NOTE ON THE MISKOLCZI THEORY

1. ABSTRACT
The Miskolczi theory of our atmosphere is summarized. The main relations of the radiative and non-radiative heat fluxes are derived. The physical control mechanisms that keep the atmospheric fluxes bound to these relations are described. The physical mechanisms of climate change that are published and that are compatible with this theory are mentioned. In an appendix the main analytical derivation of Miskolczi is explained.

2. INTRODUCTION
I have written this note in answer to a question from the KNMI, the Dutch weather institute. The new theory is so different from the usual stream of thought on climate change, that many are either confused, or revolt or embrace this theory without really having penetrated it.

3. FIXED ENERGY FLUX RATIOS
Neglect $P^0$: in a period much longer than the one month time constant of heating the 100 m top mixed ocean layer, $P$ is very small. $S_G = S_U$ when the surface radiates as a black body. See flux diagram below from [1]; the theory is published in [1,2].
Arrow joints: $OLR = S_T + E_U$, $S_U = A_A + S_T$. Only 4 of these 5 equations are independent.

Measurements: $A_A = E_D$; $2E_U = S_U$; $3E_D = 5E_U$. See the two lines in the graph taken from [2], surface temperatures range here from -40ºC to +37ºC. There is no “standard atmosphere”, these three relations hold for all climates, from the polar night to the tropical ocean afternoon.

Every point is the result of Hartcode line-by-line calculations of upward $E_U$ and downward $E_D$ energy fluxes from one of hundreds of radiosonde measurement series of humidity, temperature and pressure. With these three relations, together with the energy balances over atmosphere and surface and the known value for the net insolation $F^0$, there is only one degree of freedom left, that is the ratio $F/K$. This ratio is not at all constant; during the polar night or winter $F^0 = F = 0$, but $K$ is relatively large; there is a large horizontal non-radiative heat flux from the tropics to higher...
latitudes, dominating the climate. The 3 points on the extreme right are from desert climates where there is no water to evaporate and K is limited to sensible heat transfer.

Putting a value on the $F^0$ of 12, in units of 21 W/m² in order to get simple numbers, we have 8 equations can solve the 8 equations: 4 balances & arrow joint relations of the diagram below from [2] and 3 measurement relations of the graph above, and $F^0=12$. We get 8 values for the fluxes: $F^0=12$, OLR=12, $F+K=9$, $E_D=15$, $A_A=15$, $E_U=9$, $S_G=18$, $S_T=3$. [$P^0=0$]

![Diagram](image)

**Fig. 1.** Radiative flux components in a semi-transparent clear planetary atmosphere. Short wave downward: $F^0$ and $F$; long wave downward: $E_D$; long wave upward: OLR, $E_U$, $S_T$, $A_A$, and $S_G$; Non-radiative origin: $K$, $P^0$ and $P$.

This is a remarkable result. It means that the greenhouse factor is not a free variable: OLR is 2/3 of $S_U$. The optical density is $\ln[S_U/S_T]=\ln[18/3]=1.8$. This value, see appendix, is the same as the theoretical value resulting from Miskolczi’s new solution of the Schwarzschild-Milne radiation equation for a bounded semi-transparent atmosphere, with 1] the bottom boundary condition that the surface temperature is equal to the air temperature just above it, and 2] the top boundary condition that the downward radiation is zero, and further the assumption that 3] the radiation energy loss from the top of the atmosphere is maximal with given $S_G$. The solution is $\text{OLR}/S_G=2/[1+\tau_A+\exp(-\tau_A)]$ The exact $\tau_A$ value, 1.868, from $\text{OLR}/S_G=3/5+2/5\exp(-\tau_A)=2/[1+\tau_A+\exp(-\tau_A)]$, is equal to the long-time mean global value, 1.8688, in the NOAA-NCEP data base, the 1.8737 value in the TIGR2 radiosonde data base, and the 1.8693 value in the GAT profile.

To summarize the main features of Miskolczi’s theory:
- The heat transfer from surface to atmosphere is only by convection, not by radiation.
- We do not need to know the composition of the atmosphere.
- We do not need to subdivide atmosphere into troposphere and stratosphere.
- We do not need to differentiate between low and high latitudes; the theory holds everywhere.
- We do not need to differ between low & high clouds, only their total albedo effect matters.
- The surface temperature $T_S$ is only coupled by $S_U=\sigma T_S^4=1.5 F^0$ to the net SW absorption $F^0$.
- There is no “greenhouse gas”, no “forcing”, no “feedback”, no “climate sensitivity”.
- The cloudy sky moves to that equilibrium effective optical density whereby the net absorbed solar heat can be reradiated out into space with the minimum greenhouse effect, minimum surface temperature or maximum entropy production.
4. PHYSICAL MECHANISMS

Now there are three questions that arise:
1. What is the mechanism that makes $A_A = E_D$?
2. What is the mechanism that makes $E_U = S_U/2$?
3. What are the mechanisms that causes large climate changes such as ice ages, notwithstanding this fixation of flux relations, and notwithstanding an almost constant extraterrestrial solar flux?

4.1. The mechanism behind $A_A = E_D$

This is a consequence of the atmosphere being everywhere at LTE or Local Thermodynamic Equilibrium. We can measure a real temperature at least up to 60 km height. Because of the fact that IR absorbing gases are minor components of our atmosphere and the maximum thermal emission intensity is the emission from the surface that is in thermal contact with the atmosphere, the frequency of collisions of excited molecules that quench the excited state is many orders of magnitude higher than the inverse of the life time of the excited state. So, any emission is of purely thermal origin, any absorption cross section is therefor equal to the emission probability. The condition of LTE is necessary, but not sufficient. We need also a small enough lapse rate, that is determined by convection, [6.5 K/km] rather than by radiation [50 K/km] when there would be no convection. With this lapse rate the temperature gradient over the mean free path of a IR interacting photon is so small that $A_A$ is only 10 W/m$^2$ larger than $E_D$, the latter coming from a slightly higher and therefore colder layer.

![Graph showing $E_u, E_d, A_a$ as function of height](image)

We see in the graph above, calculated with HARTCODE from the GAT profile, that $E_D = A_A$ dominates the lower atmosphere under the cloud base, or in the turbulent mixing layer, and prevents heat transfer other than by convection $K$, and window radiation $S_T$, $E_U$ originates in the upper part of the troposphere, using the heat brought there by convection and, still higher up, by shortwave light absorption. In the graph below, we see that $E_U$ is proportional to height over almost 5 orders of magnitude. The net upward IR radiation from the atmosphere originates in the spectral lines becoming narrower with altitude, in the order of 0.1/cm per atmosphere. The typical water line width at half-maximum is at the surface [1000 hPa] 10 times wider than at 53 km [50 hPa]
4.2. The control mechanism for $E_U = S_U/2$

The answer can only be that the main “greenhouse gas”, i.e. water, is available in unlimited supply and finds its way into the atmosphere to control the flux relations.

We know from measurements; TOGA-COARE-IOP in this case, that increased latent heat flux, the main component of $K$, goes along with drying the atmosphere above the cloud base, or over the turbulent boundary layer. See http://www.cpc.noaa.gov/products/wesley/toga/toga.html

In the period between 1973 and 1997 in the tropical West-Pacific the sea surface temperature rose with only 0.4 ºC, the latent heat flux increased 15 W/m$^2$, or about 9%, the water content changed +1.3% at 1000 mB, -6.8% at 925 mB, -10% at 850 mB, and -29% at 500 mB, causing a large decrease of local atmospheric optical density. Both effects caused an increase of $E_U = F + K$ and therefore of the OLR = $S_T + E_U$. In this way the surface temperature is tightly controlled: If $S_U$ would rise with 15 W/m$^2$, then $T_S$ would rise with 2.5 ºC, 6 times more as it did. The measured differentials, albeit local in nature, are larger than can be related to increased CO$_2$ concentration, that rose in this period from 330 to 380 ppmv, or 8.6%. A relative change in upper tropospheric humidity [UTH] has a 10 times larger relative IR transparency effect as an equal relative CO$_2$ change has: $d[OLR]/d[UTH] = 0.4 \text{ W/}[m^2\%]$; $d[OLR]/d[CO_2] = 0.04 \text{ W/}[m^2\%]$. Paltridge et al.[3] find that the climate sensitivity for a CO$_2$ doubling changes from 1.6 ºC with unchanging relative humidity, down to 0.4 ºC when the currently observed drying trend is taken into account.

4.2.1. Virial Law.

We have an idea how the control mechanism works, as long as the water partial pressure is able to influence the optical density. Now we come to the attractor, or “set point” of the control whereby $E_U = S_U/2$. Miskolczi has a simple answer; “Virial Law”, that I will explain here: The Virial Law says that in a system where potential energy [gravitational energy] can be traded freely against kinetic energy [molecular kinetic energy], that the latter is always half of the former. So, for a mole of diatomic gas with kinetic energy $C_vT = 5RT/2$ and molecular mass $\mu$ at height $h$ in a
gravitational field with acceleration g, the Virial Law becomes \(5RT/2 = \mu gh/2\). For \(T=289K\), \(h\) becomes 8.44 km when \(T\) is independent of \(h\). We know that \(T\) varies along a lapse rate of \(dT = \mu g/C_p dh = \mu g/[7R/2]dh\) and we have to correct for this temperature if we express potential energy in \(T_s\), the surface temperature. The corrected height becomes \(h = R/\mu g[T_s - \mu g/[7R/2]h]\) or \(h = 1/[1 + 2/7]RT_s/\mu g\) and the Virial Law becomes, reckoned from the surface, with \(T_e\) the emission temperature of \(E_U\), \(T_e = T_s - T_s/[1 + 2/7]/5\). The “Virial” ratio \(E_U/S_U\) becomes \([T_e/T_s] = (1 - 1/5/[1 + 2/7]) = 0.508\), Miskolczi’s measured ratio is 0.5±0.1, from polar night to hot Pacific afternoon. When we correct for the 1% Argon with its \(C_v = 3R/2\) in the atmosphere, and assume that on the height where most \(E_U\) radiates from, water concentration can be neglected, the numbers 5 and 7 change to 4.98 and 6.98 in the expression for \(E_U/S_U\) and their ratio becomes 0.507, exactly the same that we derive using the exact value of \(\tau_A\), 1.868, from \(\text{OLR}/S_G = 3/5 + 2/5 \exp(-\tau_A) = 2/[1 + \tau_A + \exp(-\tau_A)]\).

The fluxes at this \(E_U/S_U = 0.507\) set point are controlled in order that the conversion of \(P^0\) into OLR proceeds with the lowest surface temperature possible, i.e. with the highest entropy production, as with all thermal dissipation. We find maximal entropy production [MEP] in all thermal dissipation processes: The temperature distribution over successive radiation screens, the turbulent flow in boiling fluids and thermal convection all yield MEP within the allowed physical constraints of the dissipative structure.

### 4.3. Climate change mechanisms

#### 4.3.1. The recent climate change

We have 9 fluxes constrained by 8 relations, so there is only one degree of freedom. In the long run, the only way to change the climate is to change \(P^0\), the net absorbed solar shortwave energy. \(P\) and \(P^0\) can be neglected on a global scale over years.

Now \(P^0 = S^0/4 (1 - \alpha)\) where \(S^0\) is the solar constant, 1368 W/m², and \(\alpha\) is the Earth albedo, 30%, and \(\alpha = \beta/2\), where \(\beta\) is the global cloud cover, 60%. So, when \(\beta\) decreases 4%, from 68 to 64%, \(\alpha\) decreases from 34% to 32%, \(1 - \alpha\) increases from 76 to 78%, and \(P^0\) grows by 78/76 = 2.6%. All fluxes scale by \(P^0\) so the surface flux has to increase also 2.6%, this needs a surface temperature increase of 2.6%/4 = 0.65%, or from 288 K to almost 290 K. The mean cloud amount indeed has changed 4% in the warming period 1986-2008:
Albedo has been measured independently by Earth-shine on the Moon to have decreased indeed about 2% from 1985 to 2004. This decrease explains all warming in this period [4].

4.3.2. Orbit excentricity
The effective insolation varies 6.6% between 3 January [min] and 4 July [max] due to the excentricity of Earth’s orbit. Paltridge [6] used this perturbation to measure a cloud cover increase of 0.009 per % insolation increase and a surface temperature increase of 0.35 K per % insolation increase. This translates into a cloud cover amplitude of 0.009*6.6*0.66=0.039, somewhat larger than the yearly amplitude of 0.03 in the ISCCP time series, and a surface temperature amplitude of 0.35*6.6=2.3K. The resulting albedo rises from 0.30 to 0.34, 1-α decreases from 70 to 66%, and F_0 diminishes by 70/76 =8.6%. But the insolation has risen 6.6%, so the net difference is 2%. All fluxes scale by F_0 so the surface flux has to decrease also 2%, this brings a surface temperature decrease of 2%/4=.5%, or 1.5 K. Measured is an increase of 2.3 K, aparently our simple calculation oversees additional feedbacks during this perturbation with a period of a year.

4.3.3. Intraseasonal oscillations
On the still shorter run, P and P_0 can influence the climate there where heat is transferred horizontally by sea currents. The sun heats with 147 W/m^2 a 100 m deep column of water, the time constant is the heat capacity, 4.2e6*100 J/m^2K, divided by the power density, 147 W/m^2, yielding 2.8e6 seconds or one month. Indeed, we see in the Pacific “Intra Seasonal Oscillations” with double this period, just like a process controller tends to oscillate with a period equal to twice the main integration time.

4.3.4. CO_2, water and cloud effects
We have seen in 4.2 that the CO_2 forcing is 0.04 W/m^2/\%*8.6%=-0.344 W/m^2, the water vapor forcing is about -0.4 W/m^2/\%*15%=-6 W/m^2, and in 4.3.1 that the cloud cover or albedo forcing is 2.6%*252 W/m^2=-6.5 W/m^2. The CO_2 effect is small in comparison to the two opposing effects of upper atmospheric drying and cloud cover decrease. It is plausible that less clouds go together with a drier atmosphere. It is clear that the CO_2 increase cannot be the major cause. There must be some other mechanism that drives both large effects. In a MEP structure, that mechanism can be a change in the physical constraints to entropy production.

4.3.5. The faint early sun paradox
In Archean times, the solar constant was 30% less than it is now. When we put the 0.009 value as relevant even then, the cloud cover was .66 - 0.009*30=39%, the albedo 18.5%, F_0 0.7[1-1.185]/[1-0.33]=85% of what it is now, and surface temperature, following Miskolczi, was 0.85^{10.25}\cdot 289=278K, just preventing the oceans from freezing. We need not to know the atmospheric composition in those times. The cloud cover feedback is so strong that it could even manage this large difference in insolation.

4.3.6. Ice ages and the PETM
The “dissipative structure” of our atmosphere is defined by that amount of sensible & latent heat transfer from surface to the TOA, E_U, that together with the IR radiation S_T from surface through the IR window can transfer the net incoming flux F_0 to the OLR with the lowest surface temperature, that is, with the highest entropy production. The rest of the solar flux is reflected without entropy production. During an ice age, the Earth is much whiter. More sunlight is reflected. The surface temperature is lower, and along the surface dissipation path the entropy production is higher. The flux relations stay the same, but F_0 and therefore all the other fluxes are smaller, and in case the solar constant has not changed, the surface temperature is lower by a relative amount of (1-α)^{1/4}. An α increase, or cloud cover increase, of 13% corresponds with a 10 °C lower surface temperature, an ice age whereby land ice in Europe reaches 50° latitude and covers the Netherlands.
During the Paleocene-Eocene Thermal Maximum the polar temperature was 20 °C, the tropical SST 36°C. This would be the result of a 13% lower albedo due to less cloud cover, a stronger trade wind, drier deserts, more sensible heat transfer horizontally to the poles. We do not know what caused the low cloud cover. What we know is a large-scale anaerobic sequestration of sulphur as FeS or pyrite during this period.

5. CLOUD COVER CHANGE

So our question changes into the following one:
What causes such large cloud cover changes of +/- 13%?
Converting latent heat or water vapor content by condensation into clouds & into sensible heat requires cloud condensation nuclei [CCN]. For deep convection [K], the efficiency of this conversion or condensation is essential. When there are more CCN, the atmosphere becomes more opaque and the heat transfer mode in the lower atmosphere shifts more into the convective mode. F₀-F, the part of the insolation that reaches the surface, decreases. Both effects bring a cooling of the surface. The upper atmosphere gets wetter, the cloud cover increases and the albedo increases. CCN are particles have at least the critical dimension, about 80-120 nm, from which a cloud water droplet can grow. Smaller droplets evaporate. There are not enough CCN for the condensation to reach equilibrium everywhere. Supersaturation is ubiquitous above the cloud base and away from clouds. Conversion of latent into sensible heat has a wide range of efficiencies, between 95 and 5%, due to more or less CCN availability. More CCN make finer droplets and whiter clouds.

5.1. Sulphuric acid

The main factor for growing CCN from the large supply of much finer particles is sulphuric acid. When atmospheric sulphuric acid increases as a result of a volcanic eruption the global temperature decreases often several tenths of a °C within a year.
There is a Nobel-prize-winning atmospheric chemist who proposed in earnest to inject millions of tons of sulphur into the stratosphere to replace the feared anthropogenic global warming by anthropogenic global cooling.
If non-sea salt sulphate [sulphuric acid, not sodium sulphate] lowers the temperature, we should find more non-sea salt sulphate in ice cores during ice ages than during interglacial periods.
Around 1990 we were most interested in the CO₂ content in polar ice cores, to prove that variations in the concentration of this gas caused ice ages, and moreover to suggest that there is a large amplification, almost ten times, going from the radiation forcing temperature increase to the total temperature increase. See the left graph below,
iahs.info/redbooks/a208/iahs_208_0029.pdf
that was used for this purpose. And indeed, there is a good correlation between CO₂ and the temperature proxy, here ice mass or heavy oxygen isotope ratio.
Later it turned out that the CO₂ concentration followed the temperature change by about 600 years, the deep mixing time of the oceans, and therefore could not be the causal factor but only an effect of changing temperatures. In the left graph with its low time resolution we cannot see this lag. We see that the insolation variation, following the 41 ky Milankovitch cycle, indeed starts the first and the last of the four thawing periods, indeed preceding them with a kiloyears lead.

The right graph, due to Legrand 1992, much less widely published, from the same time and the same ice core, concentrates on sulphate instead of on CO₂:
http://ocw.mit.edu/NR/rdonlyres/90E02A75-CAB2-4211-9764-BF347CA8F27A/0/lec08.pdf
We see that sulphate varies almost a factor of three, high in cold and low in warm periods. In the upper graph with the Deuterium isotope ratio as a temperature proxy, we see the same temperature time series. With this resolution we cannot see if the sulphate decrease is preceding or following the temperature increase. The delay should be short however, because the sulphate dissolved in the oceans does not interact here, and is even subtracted as Na₂SO₄ to get the n.s.s. value that is
relevant for the CCN production. MSA stands for [di-]methylsulphonic acid, a gaseous product from marine biota that is known to increase with temperature. Its concentrations however are more than one order of magnitude lower.

We know from experience that a major volcanic eruption causes a few years of global cooling, and that this cooling is directly connected to SO$_2$ emission, that oxidizes into sulphuric acid and cools by increasing albedo and cloud cover. The “year without summer”, 1816, with a 3 K temperature fall in Europe, snow storms in June, an 8-fold increase in grain prices and a hundred thousand deaths from famine, was caused by mount Tambora’s 1815 explosion on the island of Sumbawa. Sulphate content of Greenland ice in 1816 was measured to be 4 times higher than in preceding and following years. Journal of Geophysical Research, Vol. 100, p. 26105, determines the maximum forcing at 97% relative humidity to be 2000 W/gSO$_4^{2-}$. The accepted forcing of 2xCO$_2$ in the literature is 4W/m$^2$ or 16W/gCO$_2$ or 4ºC/gCO$_2$/m$^2$. From the graphs hereunder we can derive a sulphate forcing of 6ºC/125 ppm SO$_4^{2-}$ or 7500 ºC/gSO$_4^{2-}$. Almost 2000 times the supposed CO$_2$ effect.

It is remarkable that the sulphate content of ice cores correlates so well with the temperature proxy and that hydrosulphuric acid is a so well proven cause of climate change, but receives so little attention in the literature. During the Paleocene-Eocene Thermal Maximum, we see a strangely large sulphur isotope abundancy change, a $^{34}$S minimum, the implications of which, presumably anaerobic pyrite sequestration, remain unclear.

5.2. Galactic Cosmic Rays

There is another remarkable correlation with temperature, that has been recognized already in 1975, [5]. That is the large variation in time of galactic cosmic ray [GCR] intensity, the atmospheric production of $^{10}$Be and $^{14}$C, correlated with the variation of the number of sun spots
and change of climate. The hypothesis is that charged electric particles $[\mu^+ \text{ and } \mu^-]$ created by these very energetic protons in the lower atmosphere charge and therefore enhance the coalescence of $\text{H}_2\text{SO}_4$ pre-condensation nuclei. The correlation with climate is extraordinary on any time scale, see the three following graphs from CERN-PH-EP/2008-005: We see that only one curve does not correlate well, i.e. the infamous “hockey stick”, prominent in the IPCC TAR.

Lake Mucubaji is on 3500 m altitude in the Venezuelan Andes, where an extensive paleoclimate study has been made; [www.geo.umass.edu/climate/theses/polissar-thesis.pdf](http://www.geo.umass.edu/climate/theses/polissar-thesis.pdf)
Fig. 9: Profiles of $\delta^{18}$O from a U-Th-dated stalagmite from a cave in Oman, together with $\Delta^{14}$C from tree rings in California bristlecone pines and elsewhere, for a) the 3.4 ky period from 9.6 to 6.2 ky BP (before present) and b) the 430 y period from 8.33 to 7.9 ky BP [65].
6. CONCLUSION

We have seen that the Miskolczi theory, very different from what is the basis of current complicated climate models, and much more fundamental, excludes temperature increase by increasing greenhouse gases, their only effect being a small rise in rainfall and upper atmospheric drying. We have seen that the large climate changes in the past can be explained by changes in net insolation due to changes in cloud cover c.q. Earth albedo. These changes in cloud cover are perfectly correlated with changes in the cloud condensation nuclei change due to sulphuric acid and to galactic cosmic rays.

Twekkelo, May 25, 2010
7. APPENDIX

The derivation of the correct greenhouse function, obtained by solving the Schwarzschild/Milne radiation equations with only the surface temperature as lower boundary condition and only upward radiation as higher boundary condition, and as further constraint that the radiation out of the atmosphere itself is maximized according to the general principle of maximum entropy production, in this case the lowest greenhouse effect, is due to Ferenc Miskolczi [2]. The main parts of the derivation follow here in facsimile, to allow me to make some remarks on the main points:

dependence of the surface air temperature and the ground temperature on the total flux optical depth, (Goody and Yung, 1989; Stephens and Greenwald, 1991; McKay et al., 1999; Lorenz and McKay, 2003):

\[ t^+_d = t^+_s (1 + \bar{f}_d) / 2, \]
\[ t^+_b = t^+_s (2 + \bar{f}_d) / 2, \]

where \( t^+_d = \pi B(\bar{f}_d) / \sigma \), \( t^+_b = t^+_s + t^+_d / 2 \), and \( t^+_b = H / \sigma = OLR / \sigma \) are the surface air temperature, ground temperature, and the effective temperature, respectively. At the top of the atmosphere the net IR radiative flux is equal to the global average outgoing long wave radiation. As we have already seen, when long term global radiative balance exists between the SW and LW radiation, OLR is equal to the sum of the global averages of the available SW solar flux and the heat flux from the planetary interior.

We have seen that in a semi-transparent atmosphere the surface upward radiation is \( B_\odot = \sigma T^4 / \pi \), and the upper boundary condition at the top of the atmosphere is the zero downward IR radiance. The upward and downward hemispheric mean radiance at the upper boundary using the general classic solution of the plane-parallel radiative transfer equation and the isotropy approximation are:

\[ \bar{\tau}^+(0) = B_\odot e^{-3 \bar{f}_d / \pi} + \frac{3}{2} \int B(\bar{f}') e^{-3 \bar{f}' / 2} d\bar{f}', \]  

and

\[ \bar{\tau}^-(0) = 0. \]

Putting Eq. (B1) and Eq. (B2) into the \( H(\bar{f}) = \pi (\bar{\tau}^+ - \bar{\tau}^-) \) equation, and substituting the source function with \( B(\bar{f}) = 3H(\bar{f}) / (4\pi) + B_\odot \) in the upward hemispheric mean radiance we get:

\[ \frac{H}{\pi} = B_\odot e^{-3 \bar{f}_d / \pi} + \frac{3}{2} \int_0^{\bar{f}_d} \frac{3H}{4\pi} \bar{f} e^{-3 \bar{f}' / 2} d\bar{f}' + \frac{3}{2} \int_0^{\bar{f}_d} B_\odot e^{-3 \bar{f}' / 2} d\bar{f}'. \]  

The two definite integrals in the second and third terms of the right hand side of Eq. (B3) must be evaluated:

\[ \frac{3}{2} \int_0^{\bar{f}_d} \frac{3H}{4\pi} \bar{f} e^{-3 \bar{f}' / 2} d\bar{f}' = -\frac{H}{4\pi} \left( 2e^{-3 \bar{f}_d / 2} - 2 + 3\bar{f}_d e^{-3 \bar{f}_d / 2} \right), \]

and

\[ \frac{3}{2} \int_0^{\bar{f}_d} B_\odot e^{-3 \bar{f}' / 2} d\bar{f}' = B_\odot (1 - e^{-3 \bar{f}_d / 2}). \]

After putting back Eqs. (B4) and (B5) into Eq. (B3) we get:

\[ \frac{H}{\pi} = B_\odot e^{-3 \bar{f}_d / \pi} + \frac{H}{4\pi} \left( 2e^{-3 \bar{f}_d / 2} - 2 + 3\bar{f}_d e^{-3 \bar{f}_d / 2} \right) + B_\odot (1 - e^{-3 \bar{f}_d / 2}). \]
Rearranging Eq. (B6) and using the \( \tilde{\tau}_d = (3/2) \tilde{\tau}_d \) notation for the total flux optical depth, \( \pi B_0 \) can be expressed as:

\[
\pi B_0 = \frac{\frac{\mu}{2} \left[ 1 + \tilde{\tau}_d e^{-\tilde{\tau}_d} + e^{\tilde{\tau}_d} \right] - \pi B_C e^{-\tilde{\tau}_d}}{1 - e^{\tilde{\tau}_d}}.
\]  

(B7)

This \( B_0 \) in the \( B(\tilde{\tau}) = 3 H \tilde{\tau} / (4 \pi) + B_0 \) equation will give the general form of the source function profile:

\[
\pi B(\tilde{\tau}) = \frac{\frac{\mu}{2} \left[ 1 + (\tilde{\tau}_d - \tilde{\tau}_A) e^{\tilde{\tau}_d} + e^{-\tilde{\tau}_d} \right] - \pi B_C e^{-\tilde{\tau}_d}}{1 - e^{\tilde{\tau}_d}}.
\]  

(B8)

Applying the \( T_A = \exp(-\tilde{\tau}_d) \), \( A = 1 - T_A \), and \( f = 2/(1 + \tilde{\tau}_d + T_A) \) notations, Eq. (B8) will become identical with Eq. (21). The semi-infinite solution may be obtained exactly in the same way, but substituting \( \tilde{\tau}_d \) with infinity in Eq. (B1), or simply by making these substitutions in Eq. (B8).

The most efficient cooling of the clear atmosphere requires a total optical depth that maximizes \( B_0 \). The derivative of Eq. (B7) with respect \( \tilde{\tau}_d \) may be expressed as:

\[
\pi \frac{dB_0}{d\tilde{\tau}_d} = \frac{d}{d\tilde{\tau}_d} \left[ \frac{\pi B_C e^{\tilde{\tau}_d} - \frac{\text{OLR}}{2} \left[ 1 + \tilde{\tau}_d + e^{\tilde{\tau}_d} \right]}{e^{\tilde{\tau}_d} - 1} \right].
\]  

(B9)

From Eq. (B9) follows that:

\[
\frac{\pi B_C e^{\tilde{\tau}_d} - \frac{\text{OLR}}{2} \left[ 1 + \tilde{\tau}_d + e^{\tilde{\tau}_d} \right]}{(e^{\tilde{\tau}_d} - 1)^2} = 0.
\]  

(B10)

From Eq. (B10), assuming \( \tilde{\tau}_d > 0 \) we get:

\[
\pi B_C = \text{OLR} \frac{1 + \tilde{\tau}_d + e^{-\tilde{\tau}_d}}{2} = \frac{\text{OLR}}{f}.
\]  

(B11)

**Classical solution**

Equations 16 & 17 are the classic 1922 Eddington solutions of the Schwartzschild-Milne equations, originally conceived to explain the enormous “greenhouse effect” in the Sun’s atmosphere, where the inside temperature is a thousand times the surface temperature. The Sun has no surface, and the sun’s atmosphere is a plasma; its optical density is many orders of magnitude higher than that of our atmosphere, therefore an infinite approach does not bring large errors. But on Earth it cannot use infinity, otherwise \( \tau_A \) would be infinite. So it uses an artificial upper boundary, that is the effective height with temperature \( t_E \) where the OLR radiates from. This is at about 5 km height. This is not the top of the atmosphere, not the height whereby the downward radiation is zero, and therefore not a physically correct boundary condition. Another unphysical result is that the surface temperature \( t_0 \) becomes about 20 °C higher than the air temperature \( t_a \) just above it. The Keith-Trenberth scheme, or “US standard atmosphere” shows this discrepancy in that the absorbed part \( A_A \) of the upwelling radiation from the surface is some 25 W/m² more than the downwelling radiation \( E_D \). This unphysical assumption helps to come to a 5 W/m² forcing at the surface due to a CO₂ doubling, but it is physically not correct, and is probably a main cause of error in the derivation of the “climate sensitivity” from the climate models. It leads to a 25 W/m² unacceptable underestimation of the window radiation. The K/T scheme only holds for the “mean global” temperatures and fluxes. There exists nowhere on Earth a “mean” and consistent set of temperatures and fluxes. We need a solution that describes the atmospheric physics correctly from the polar winter all the way to the equatorial afternoon. Only then we can take a mean value of the flux ratios, not of the fluxes themselves.
Miskolczi’s solution

Equation (B1) has only one surface temperature and radiation flux: \( S_U = S_G = \sigma t_A^4 = \sigma t_A^4 \) as lower boundary condition, assuming correctly that the surface has an emissivity near to one and there is a temperature continuity between air and surface.

Equation (B2) has no infinity problem at the top of the atmosphere, so needs no artificial assumption for an upper boundary, but can use the real the boundary condition, \( I = 0 \), zero downward flux, and therefore zero absorbed upward flux.

The derivation now proceeds the classic way, described in many handbooks. From equation (B8) we can derive directly that \( E_U = \pi B_0^2 / A \) where \( A = 1 - e^{-\tau_A} \), the absorption of the semi-transparent atmosphere. This is the value that implies the equilibrium greenhouse effect. Note that \( E_U \) is brought into the atmosphere by absorption of shortwave radiation and by non-radiative heat transfer from the surface, and not by surface radiation!

Equation (B9) sets the derivative of \( B_0 \) to \( \tau_A \) to zero. This is an important extra condition. It assumes that our atmosphere, as all thermal dissipative structures do, organizes itself for minimal temperature difference given the flux, or for maximal flux given the temperature difference, or for maximal entropy production \([\text{flux}/T_{\text{low}} - \text{flux}/T_{\text{high}}]\). In the derivation, it is not important how the atmosphere does this, only that there are enough degrees of freedom available, such as the supply of water into the atmosphere that has a large influence on \( \tau_A \) and therefore can control \( \tau_A \).

With this condition the greenhouse factor is determined: \( \text{OLR}/S_G = 2/[1 + \tau_A + \exp(-\tau_A)] \).

The strong point for this purely theoretical derivation, that uses no climate parameters and no material constants at all, gives a \( \tau_A \) derived from \( S_G \) and OLR values that is exactly equal to the measured \( \tau_A \), either the TIGR value or the completely independent NOAA/NCEP value, with a precision into the third decimal. \( \tau_A \) has the value of 1.86. This is and must be a global effective value, because it implies essential atmospheric water vapor transport and heat transport by horizontal convection from equator to poles that keeps global \( \tau_A \) on this value!

We see that if \( \tau_A \) becomes large, \( 2/[1 + \tau_A + \exp(-\tau_A)] \) becomes the old solution \( 2/[1 + \tau_A] \) in equation (16). This is correct for the Sun. If \( \tau_A \) goes to zero, we have no greenhouse effect, \( S_U = S_G = \text{OLR} \), correct for the Moon.

Below a visual reminder: All this algebra is but a terrible simplification. All physics is. White is atmospheric water vapor emission as seen by a satellite. \( \tau_A \) is very much a local variable indeed!
8. REFERENCES


